# Finite Dimensional Integrable Systems in Geometry and Mathematical Physics

Friedrich-Schiller-Universität Jena, Germany
26th – 29th July 2011

Programme
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# Programme

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TUESDAY — 26TH JULY 2011

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| 08:30 – 09:00 | Registration |
| 09:00 – 09:15 | Opening |
| 09:15 – 10:10 | **Michael Bialy**<br>*Polynomial integrals and Rich quasi-linear systems of conservation laws* |
| | coffee break |
| 10:35 – 11:30 | **Jan-Cees van der Meer**<br>*Perturbed harmonic oscillators and Keplerian systems, bifurcations and singularities* |
| 11:35 – 12:30 | **Peter Topalov**<br>*Geodesically compatible metrics* |
| | lunch break |
| 14:00 – 14:55 | **Eva Miranda**<br>*From action-angle coordinates to Geometric Quantization and back* |
| | coffee break |
| 15:20 – 16:15 | **Daniele Sepe**<br>*Lagrangian bundles and affinely flat geometry* |
| 16:20 – 17:15 | **Federico Zullo**<br>*Separation of variables, Bäcklund transformations and "spectrality property" for finite dimensional integrable Hamiltonian systems* |

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WEDNESDAY — 27TH JULY 2011

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| 09:00 – 09:55 | **Boris Kruglikov**<br>*The gap phenomenon in the dimension study of finite type systems* |
| | poster session & coffee break |
| 10:35 – 11:30 | **Anton Izosimov**<br>*Stability in bihamiltonian systems and the Manakov top* |
| 11:35 – 12:30 | **Chris Athorne & Matthew England**<br>*Generalised elliptic functions* |
| | lunch break |
| 14:00 – 14:55 | **Andrey Mironov**<br>*On polynomial integrals of a natural mechanical system on a two-dimensional torus* |
| | coffee break |
| 15:20 – 16:15 | **Andrzej J. Maciejewski & Maria Przybylska**<br>*Differential Galois obstructions to integrability. An overview.* |
| 16:20 – 17:15 | **Boris Zhilinskii**<br>*From Hamiltonian monodromy to lattice defects and spiral phyllotaxis* |

# Programme

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Thursday — 28th July 2011

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|---|---|
| 09:00 – 09:55 | **Galliano Valent**<br>*1+3 decomposition of quadratic Killing-Stäckel tensors in curved spaces* |
| | coffee break |
| 10:20 – 11:15 | **Yuri Fedorov**<br>*Separation of variables and explicit solutions*<br>*of some integrable systems with cubic integrals* |
| 11:20 – 12:15 | **Kazuyoshi Kiyohara**<br>*A class of Hermitian manifolds with integrable geodesic flows* |
| | lunch break |
| 14:00 | excursion to Weimar |
| 18:00 | conference dinner |

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Friday — 29th July 2011

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| 09:00 – 09:55 | **Jeandrew Brink**<br>*Killing tensors on SAV Spacetimes and their implications*<br>*for Mapping Spacetime observationally* |
| | coffee break |
| 10:20 – 11:15 | **Vsevolod Shevchishin**<br>*Superitegrable surface metrics admitting one linear and one cubic integral* |
| 11:20 – 12:15 | **David M. J. Calderbank**<br>*H-projective invariance in Kähler geometry* |
| | lunch break |
| 14:00 – 14:55 | **Heinz Hanßmann**<br>*The degenerate C. Neumann system* |
| | coffee break |
| 15:20 – 16:15 | **Daisuke Tarama**<br>*Explicit Construction of K3 Dynamical Models and their Hamiltonian Monodromy* |
| 16:20 – 17:15 | **Stefan Rosemann**<br>*Yano-Obata conjecture for holomorph-projective transformations* |

# Abstracts

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**Chris Athorne & Matthew England** University of Glasgow, United Kingdom

*Generalised elliptic functions*

We consider multiply periodic, or Abelian, functions defined with respect to the period matrices associated with classes of algebraic curves. We realise them as generalisations of the Weierstrass P-function using two different approaches. These functions arise naturally as solutions to some of the important equations of mathematical physics, and so a greater understanding of their properties is desirable. The differential equations, addition formulae, and applications of the functions have all been recent topics of study with progress following both from new theory, and advancement of efficient symbolic computation techniques.

The first approach discussed sees the functions defined as logarithmic derivatives of the sigma-function. We can then make use of known properties of the sigma function to derive power series expansions and in turn the properties mentioned above. This approach has been extended to a wide range of non hyperelliptic and higher genus curves and some recent results will be discussed. The second approach defines the functions algebraically, after first modifying the curve into its equivariant form. This approach allows the use of representation theory to derive a range of results at lower computational cost. This theory has been recently developed for a selection of hyperelliptic curves with general hyperelliptic and non-hyperelliptic cases now under consideration.

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**Michael Bialy** Tel-Aviv University, Israel

*Polynomial integrals and Rich quasi-linear systems of conservation laws*

In this talk I shall discuss recent results (some of them are joint with Andrey Mironov) on polynomial integrals for geodesic flows, and for systems with potentials. These results are obtained using the observation that the quasi-linear system on the coefficients of the integral has the remarkable structure of being Rich or Semi-Hamiltonian. In the talk I shall discuss this notion as well as useful facts of the Theory of Rich systems.

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**Jeandrew Brink** National Institute of Theoretical Physics, Stellenbosch, South Africa

*Killing tensors on SAV Spacetimes and their implications for Mapping Spacetime observationally*

The Stationary Axisymmetric Vacuum (SAV) field equations for the gravitational field around an arbitrarily shaped compact object in General Relativity are integrable. The solutions can be expressed in terms of an Ernst potential and are uniquely identified by means of their multipole moments. Several solution generation techniques exist to generate the explicit metrics. The nature of geodesic flow on these manifolds however has not been fully resolved. Observationally gravitational wave (GW) detectors such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Laser Interferometer Space Antenna (LISA) and radio telescopes such as the Square Kilometre Array (SKA) record the position of a particle (neutron star) probing the geodesic structure of the spacetime. For type D spacetimes such as Kerr, which admits a second order Killing tensor, the geodesic problem is solved. This feature is used extensively in all data analysis strategies implemented to date. For general SAV spacetimes beyond Type D however, the nature of geodesic motion is not well understood.

In this talk I give an overview of the general mathematical framework for mapping spacetime around a quiescent compact objects, within the paradigm of Extreme Mass Ratio Inspirals (EMRI's). In particularly, I focus on new results with respect to the existence and evaluation of 4th order Killing tensors on general SAV spacetimes, using the Zipoy Voorhees metric as an example. Finally I comment on key mathematical aspects that have yet to be resolved in order to make an algorithm for mapping spacetime tractable experimentally. Such an algorithm would allow us to probe some of the very tenets (such as the no hair theorems) that undergird our understanding of gravity and GR.

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**David M. J. Calderbank** University of Bath, United Kingdom

*H-projective invariance in Kähler geometry*

In Kähler geometry, it has recently been observed that the linear differential equation governing H-projectively equivalent Kähler metrics (i.e., with common H-planar curves) essentially coincides with the differential equation for so-called hamiltonian 2-forms on a Kähler manifold. Consequently, results and techniques for studying these equations, which had before been developed separately, can be combined.

In one direction, Topalov's work on quadratic integrals provides a conceptual explanation for some of the features of hamiltonian 2-forms. In the other, local and global classification results for hamiltonian 2-forms obtained by Apostolov et al. yield corresponding classifications for H-projectively equivalent Kähler metrics.

In this talk, I will focus on the underpinning conceptual framework of H-projective structures (developed by Yoshimatsu et al.) and sketch the above ideas in this context. I will emphasise the role of toric geometry and the geometry of the projective line in developing the theory further.

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**Yuri Fedorov** Universitat Politècnica de Catalunya, Barcelona, Spain

*Separation of variables and explicit solutions of some integrable systems with cubic integrals*

In many classical algebraic integrable systems, e.g., the Clebsch integrable case of the Kirchhoff equations or the celebrated Kovalevskaya top, the genus of the underlying curve (the spectral curve of the corresponding Lax representation) is bigger than the number of degrees of freedom (and the dimension of generic invariant tori (two)).

In such cases the curve has an involution, which extends to its Jacobian variety. Then, as a rule, the complex invariant manifolds of the systems turn out to be 2-dimensional Abelian (Prym) subvarieties of the Jacobians, whose real parts give the invariant tori.

However, there is an ample class of integrable systems for which the above is no more true, in particular the Goryachev integrable case of the Kirchhoff equations or a similar system on $T^*S^2$ discovered by Dullin and Matveev. Namely, one can show that these systems are reduced to quadratures containing trigonal genus 3 curves, which do not admit an involution, while the invariant tori are 2-dimensional. The dynamics of the systems is thus restricted to two-dimensional Wirtinger strata of the three-dimensional Jacobi varieties of the trigonal curves. This pathological property puts away these systems from being algebraic integrable, and their solutions are not meromorphic functions of the complex time.

We will present the quadratures and the explicit solutions of the systems in terms of the Klein-Weierstraß sigma-functions restricted to the two-dimensional strata and give a complete description of their complex singularities.

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**Heinz Hanßmann** Universiteit Utrecht, Netherlands

*The degenerate C. Neumann system*

The C. Neumann system describes a particle on the $n$-sphere under the influence of a potential that is a quadratic form. The degeneracy in question is that the quadratic form has eigenvalues with multiplicity, $l+1$ of which are different. Each group of $m$ equal eigenvalues leads to an $\mathrm{O}(m)$-symmetry in configuration space. The combined symmetry group $G$ is a direct product of $l+1$ such factors, and its cotangent lift has an equivariant Momentum mapping. Regular reduction leads to the Rosochatius system on the $l$-sphere, which has the same form as the Neumann system albeit for an additional effective potential.

For singular values of the Momentum mapping we obtain this same form with part of the effective potential switched off. To understand how these reduced systems fit together we use singular reduction to construct an embedding of the reduced Poisson space $T^*S^n/G$ into $\mathbb{R}^{3l+3}$. This allows to describe the global geometry, in particular the bundle structure that appears as a result of the superintegrability of the system.

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**Anton Izosimov** Loughborough University, United Kingdom / Moscow State University, Russia

*Stability in bihamiltonian systems and the Manakov top*

One of the most important questions when studying a (hamiltonian) dynamical system is the stability of trajectories, in particular, of fixed points. This problem is highly non-trivial in general. However, if a system possesses some additional integrals, everything is simplified, since these integrals are good candidates to be Lyapunov functions. In particular, if a system is completely integrable, we can usually tell for general position fixed points whether they are stable or not. More precisely, we can do so for so-called non-degenerate fixed points. In the non-degenerate case the answer is given in terms of the type of a point: if a point is pure elliptic, then it is stable, otherwise it is unstable (assuming that out system is non-resonant). Still non-degeneracy and type investigation involves heavy calculations if the number of degrees of freedom is high.

We approach the case when a system possesses a bihamiltonian structure. We show that for such systems stability problem can be solved in terms of linear parts of Poisson brackets of the corresponding Poisson pencil. As an example, we consider the Manakov top, i. e. the multi-dimensional free rigid body. It is well known, that for three-dimensional free rigid body the rotations around the shortest and the longest axes of symmetry are stable, while the rotation around the middle axis is unstable. Using the bihamiltonian techniques, we provide an analogous result for a multidimensional body.

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**Kazuyoshi Kiyohara** Okayama University, Japan

*A class of Hermitian manifolds with integrable geodesic flows*

In the paper AMS Memoirs 130/619 I studied a certain class of Kaehler manifolds whose geodesic flows are integrated in a similar way as that of the complex projective space with the Fubini-Study metric. They are called Kaehler-Liouville manifolds and I clarified local and global structures of them under certain nondegeneracy conditions.

In that theory the Kaehler condition plays a significant role for the determination of the structures. However, the Kaehler condition itself is a priori unrelated to the integrability of the geodesic flows. Moreover, starting from a Kaehler-Liouville manifold, one can easily meet Hermitian (non-Kaehler) manifolds with integrable geodesic flows. Therefore it would be natural to study Hermitian manifold (without the Kaehler condition) whose geodesic flows are integrable in the same way as those of Kaehler-Liouville manifolds.

In this talk I will show some results on the construction and the classification problems of such Hermitian manifolds (called Hermite-Liouville manifolds), mainly in the case where the underlying manifold is the complex projective space. I will also mention h-projective equivalence and "PQ-equivalence" of Topalov, together with some related works by several authors which I recognized recently, in this framework.

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**Boris Kruglikov** University of Tromsø, Norway

*The gap phenomenon in the dimension study of finite type systems*

In various geometric problems, when the maximal solution space is finite-dimensional, it is unique. Another important phenomenon is that the submaximal case has a dimensional gap with the maximal case. I will illustrate this with many examples and show one general result behind this observation. I will speculate on relation of this with integrability.

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**Andrzej J. Maciejewski & Maria Przybylska** University of Zielona Góra, Poland

*Differential Galois obstructions to integrability. An overview.*

Last three decades gave a profound progress into the problem of dynamical systems. The great impulse has been given by elegant Ziglin theory, who explained a mysterious relation between breaching of solutions as functions of the complex time and the integrability.

In the Ziglin theory necessary conditions for the integrability are expressed in terms of properties of the monodromy group of variational equations along a particular solutions of the considered system. Later on the Ziglin theory has found a very nice algebraic generalisation. Instead of the monodromy group, the differential Galois group is considered. It is appeared that this approach gives the strongest and effective tool for study integrability. We overview our results of applications of the differential Galois theory to a study of different types integrability of ordinary differential equations. Among others we discuss integrability in the Liouville sense, non-commutative integrability, partial and super-integrability, B-integrability and Jacobi integrability.

In the second part, we overview our main results concerning integrability of homogeneous and non-homogeneous potentials as well as we present integrability analysis of several famous systems.

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**Jan-Cees van der Meer** Technische Universiteit Eindhoven, Netherlands

*Perturbed harmonic oscillators and Keplerian systems, bifurcations and singularities*

In my talk I will report about several results which are for the larger part joint work with Sebastian Ferrer and co-workers of the University of Murcia. This work concerns bifurcations of relative equilibria in parameter dependent Hamiltonian perturbations of 4-DOF isotropic oscillators with toroïdal symmetry. These Hamiltonian systems are studied using constructive geometric reduction in stages. That is, reduced phase spaces are obtained through the construction of the orbit space for the different $S^1$ symmetries by the computation of invariants. On a particular set of reduced phase spaces these systems coincide with normalized perturbed Keplerian systems. The relative equilibria are obtained by considering the momentum polytope for the toroïdal symmetry and the stationary points of the final reduced system. This reduced system can be represented as a 1-DOF system on the final reduced phase space which is a semi-algebraic variety in $\mathbb{R}^3$. Bifurcation scenarios will be shown for some particular choices of the parameters.

Like Poisson manifolds also orbit spaces and reduced phase spaces allow a stratification with symplectic leaves for the induced Poisson structure on the orbit space. This stratification relates relative equilibria to singularities of the Poisson structure. Especially Hamiltonian Hopf bifurcations can only occur at singularities of the reduced Poisson structure of type $\mathrm{sl}(2,\mathbb{R})$. These phenomena will be illustrated in the geometric analysis of the above systems.

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**Eva Miranda** Universitat Politècnica de Catalunya, Barcelona, Spain

*From action-angle coordinates to Geometric Quantization and back*

A choice of polarization is a key ingredient in the Geometric Quantization scheme. A Lagrangian foliation gives a real polarization. It is a well-known result due to Snyaticki that in the case this regular Lagrangian is a fibration (with Hausdorff leaf space), the geometric quantization (à la Kostant) is given by its Bohr-Sommerfeld leaves. Under compactness assumptions, the set of Bohr-Sommerfeld leaves is discrete and can be determined via action-angle coordinates (Guillemin-Sternberg). In this talk, we will extend these results to the singular setting and explain some of our current projects. One of the main ingredients is the study of normal forms and action-angle coordinates with singularities for integrable systems in the symplectic and Poisson context.

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**Andrey Mironov** Sobolev Institute of Mathematics, Novosibirsk, Russia

*On polynomial integrals of a natural mechanical system on a two-dimensional torus*

We shall show that if a natural mechanical system defined on a two-dimensional torus and having a real analytic potential possesses a polynomial integral of odd degree in momenta, then the leading coefficients in the momenta satisfy two identities of a special form. We also show that if the system possesses an integral of the fifth degree in momenta, then there exists an integral of the first degree in momenta.

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**Stefan Rosemann** Friedrich-Schiller-Universität Jena, Germany

*Yano-Obata conjecture for holomorph-projective transformations*

The basic geometric structure in holomorph-projective geometry is the family of holomorphically-planar curves that are associated to a given Kaehler metric. Such curves can be viewed as some generalisation of geodesics on Kaehler manifolds.

In this context, one question of interest is if the group of all holomorph-projective transformations (i. e., all bi-holomorphic mappings that preserve the set of all holomorphically-planar curves) is really bigger than the group of holomorphic isometries of the Kaehler metric. Together with V. S. Matveev, we have proven a classical conjecture attributed to Yano and Obata: On a compact Kaehler manifold which cannot be covered by the complex projective space with (a multible of the) Fubini-Study metric, the connected components of the group of holomorph-projective transformations and holomorphic isometries coincide.

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**Daniele Sepe** University of Leicester, United Kingdom

*Lagrangian bundles and affinely flat geometry*

Lagrangian fibrations arise naturally in various areas of symplectic geometry, including Hamiltonian mechanics and mirror symmetry. Their topology and geometry encode information about the dynamics of completely integrable Hamiltonian systems, which are characterised by the existence of sufficiently many distinct constants of motion. These fibrations are entwined with affinely flat geometry, which studies manifolds equipped with an atlas whose changes of coordinates are constant affine transformations of $R^n$. In this talk, I shall illustrate how deep this relation goes by concentrating on the case of Lagrangian bundles, i. e. when all fibres are diffeomorphic. As a consequence of the Liouville-Mineur-Arnold theorem, the fibres and base space of a Lagrangian bundle are affinely flat manifolds. The affinely flat geometry of the former allows to classify these bundles topologically by constructing universal Lagrangian bundles. However, this classification is not sufficient to solve the problem of constructing an appropriate symplectic form on the total space of bundles which topologically "look" Lagrangian, as there is a cohomological obstruction class constructed by Dazord and Delzant. Using the topology of universal Lagrangian bundles, I shall describe this cohomology class in terms of an important affine invariant of the base space, called the radiance obstruction, introduced by Goldman, Hirsch and Smillie in the '80s. Time permitting, I will illustrate the theory with some examples.

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**Vsevolod Shevchishin** Ruhr-Universität Bochum, Germany

*Superitegrable surface metrics admitting one linear and one cubic integral*

We give a complete description of all local metrics on surfaces admitting one integral linear in momenta and one integral cubic in momenta. We also show that some of these metrics extend to global metrics on $S^2$. This gives a new example of superintegrable metrics on the sphere $S^2$ with integrals polynomial in momenta.

(joint work with Vladimir Matveev)

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**Daisuke Tarama** Kyoto University, Japan

*Explicit Construction of K3 Dynamical Models and their Hamiltonian Monodromy*

This talk deals with an explicit construction of integrable systems on elliptic K3 surfaces and their Hamiltonian monodromy. The construction is performed through the Weierstraß normal form. The real structure of the integrable system will be explained. Using deformation of two-dimensional simple singularities, some concrete examples are constructed. In particular, there is given an example of K3 dynamical model with the identity monodromy matrix around 12 elementary singular loci.

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**Peter Topalov** Boston University, USA

*Geodesically compatible metrics*

I will discuss the problem of existence of commutative integrals for a class of Riemannian metrics. These metrics appear in pairs and enjoy a special property of mutual compatibility of their geodesics. Several old and new examples will be discussed.

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**Galliano Valent** LPTHE, Université Paris VI, France

*1+3 decomposition of quadratic Killing-Stäckel tensors in curved spaces*

Let us consider a 4 dimensional metric with a single Killing vector $K = \partial_t$ of the form

$$g = \frac{\epsilon}{V}(dt+\theta)^2 + V\,(\gamma_{ij}\,dx^i\,dx^j) \qquad \epsilon = \pm 1 \qquad i,j = 1,2,3$$

When looking for a Killing-Stäckel tensor $S$ such that $\mathcal{L}_K S = 0$, using a particular (non-canonical) transformation in phase space reduces the construction of $S$ to a system of PDE immersed into the three dimensional metric $\gamma$.

This approach was successfully applied to the euclidean Multi-Centre metrics and gave, in a systematic way, all the metrics with Liouville-integrable geodesic flow. A few items will be discussed:

1. The relation between integrability and the euclidean Petrov type

2. The usual difficulty, linked to local diffeomorphisms, to identify properly the metrics according to their isometries.

3. The emergence of some W algebras.

In the more physical case of Ernst equation for axisymmetric metrics, despite the many known examples of integrable geodesic flows, a systematic analysis seems still to be lacking.

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**Boris Zhilinskii** Université du Littoral, Dunkerque, France

*From Hamiltonian monodromy to lattice defects and spiral phyllotaxis*

After short discussion of fractional monodromy [1] as a natural generalization of Hamiltonian monodromy inspired by molecular applications and the relation between monodromy of dynamical integrable systems and defects of regular lattices and patterns [2], I will describe recently suggested interpretation [3,4] of spiral phyllotaxis well known in botany in terms of lattice defects and monodromy associated to integrable dynamical system. The question about the choice of a simplest dynamical integrable model reproducing the pattern and defects characteristic for spiral phyllotaxis will be discussed from the physical point of view.

[1] N. N. Nekhoroshev, D.A. Sadovskii B. I. Zhilinskii, Fractional Hamiltonian monodromy. Ann. Henri Poincare. 7, 1099–1211 (2006).

[2] B. I. Zhilinskii, Hamiltonian monodromy as lattice defect. in: Topology in Condensed Matter, (Springer Series in Solid-State Sciences, Vol. 150), 2006, pp. 165-186.

[3] B. I. Zhilinskii, Quantum monodromy and pattern formation, J.Phys. A Math. Gen. 43, 434033 (2010).

[4] B. Zhilinskii, Monodromy and Complexity of Quantum Systems. in: The Complexity of Dynamical Systems. Ed. J.Dubbeldam, K. Green, and D. Lenstra, WILEY, Weinheim, 2011, pp. 159-181

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**Federico Zullo** Università degli Studi “Roma Tre”, Rome, Italy

*Separation of variables, Bäcklund transformations and “spectrality property” for finite dimensional integrable Hamiltonian systems*

The Bäcklund transformations for finite dimensional classical integrable systems representable in the Lax form are considered. The links with Hamiltonian dynamics, separation of variables and solutions of the Hamilton Jacobi equation of the system are presented. In this respect the so-called “spectrality property” introduced by Kuznetsov and Sklyanin about ten years ago plays a fundamental role. It is shown how the transformations can lead both to an exact time discretization of the continuous system and to the general solution of the equations of motion. Some explicit examples and pictures are also given.

# Participants

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**Akulenko, Leonid** (Institute for Problems in Mechanics of the Russian Academy of Science, Moscow)
**Athorne, Chris** (University of Glasgow, United Kingdom)
**Bialy, Michael** (Tel-Aviv University, Israel)
**Brink, Jeandrew** (National Institute of Theoretical Physics, Stellenbosch, South Africa)
**Calderbank, David M.J.** (University of Bath, United Kingdom)
**England, Matthew** (University of Glasgow, United Kingdom)
**Fedorov, Yuri** (Universitat Politècnica de Catalunya, Barcelona, Spain)
**Fedorova, Aleksandra** (Friedrich-Schiller-Universität Jena, Germany)
**Geyer, Marisa** (Stellenbosch University, South Africa)
**Hanßmann, Heinz** (Universiteit Utrecht, Netherlands)
**Izosimov, Anton** (Loughborough University, United Kingdom / Moscow State University, Russia)
**Karadzhov, Yuri** (Institute of Mathematics, NAS, Ukraine)
**Kiyohara, Kazuyoshi** (Okayama University, Japan)
**Kotov, Alexei** (University of Tromsø, Norway)
**Kruglikov, Boris** (University of Tromsø, Norway)
**Leshchenko, Dmytro** (Odessa State Academy of Civil Engineering and Architecture, Ukraine)
**Maciejewski, Andrzej J.** (University of Zielona Góra, Poland)
**Malika, Bennaoum** (Université D'Oran, Algeria)
**Matveev, Vladimir S.** (Friedrich-Schiller-Universität Jena, Germany)
**Michel, Jean-Philippe** (University of Luxembourg)
**Miranda, Eva** (Universitat Politècnica de Catalunya, Barcelona, Spain)
**Mironov, Andrey** (Sobolev Institute of Mathematics, Novosibirsk, Russia)
**Mirzaei, Reza** (Imam Khomeini International University, Qazvin, Iran)
**Morozov, Oleg** (University of Tromsø, Norway)
**Przybylska, Maria** (Institute of Physics, University of Zielona Góra, Poland)
**Pucacco, Giuseppe** (University of Rome "Tor Vergata", Italy)
**Rachinskaya, Alla** (Odessa National Mechnikov University, Ukraine)
**Richter, Christian** (Friedrich-Schiller-Universität Jena, Germany)
**Rosemann, Stefan** (Friedrich-Schiller-Universität Jena, Germany)
**Safdar, Muhammad** (National University of Sciences and Technology, Islamabad, Pakistan)
**Sakai, Hironori** (National Cheng Kung University, Tainan City, Taiwan)
**Salehi, Karim** (University of Kurdistan Hewler, Erbil, Iraq)
**Schöbel, Konrad** (Friedrich-Schiller-Universität Jena, Germany)
**Sepe, Daniele** (University of Leicester, United Kingdom)
**Shevchishin, Vsevolod** (Ruhr-Universität Bochum, Germany)
**Tarama, Daisuke** (Kyoto University, Japan)
**Tonkonog, Dmitry** (Moscow State University, Russia)
**Topalov, Peter** (Boston University, USA)
**Triadis, Dimetre** (La Trobe University, Victoria, Australia)
**Valent, Galliano** (LPTHE, Université Paris VI, France)
**van der Meer, Jan-Cees** (Technische Universiteit Eindhoven, Netherlands)
**Winther, Hendrik** (University of Tromsø, Norway)
**Zhilinskii, Boris** (Université du Littoral, Dunkerque, France)
**Zinkevich, Yanina** (Odessa State Academy of Civil Engineering and Architecture, Ukraine)
**Zullo, Federico** (Università degli Studi "Roma Tre", Rome, Italy)